A Chaotic Approach of Differential Evolution Optimization Applied to Loudspeaker Design Problem

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Abstract — Differential evolution (DE) algorithms are a family of evolutionary optimization techniques that use a rather greedy and less stochastic approach to problem solving, when compared to classical evolutionary algorithms. This paper proposes a new approach to solve electromagnetic design problems that combines the DE algorithm with a generator of chaos sequences. This approach is tested on the design of a loudspeaker model with seventeen degrees of freedom, for showing its applicability to electromagnetic problems. The results show that DE algorithm with chaotic sequences presents better, or at least similar, results when compared to the standard DE algorithm and other evolutionary algorithms available in the literature.

I. INTRODUCTION

Recent literature contains several optimization metaheuristic algorithms based on evolutionary approaches. Differential Evolution (DE) is a population-based and direct stochastic search algorithm with simple yet powerful and straightforward features that make it attractive for numerical optimization. DE uses a combining of simple arithmetic operators with the classical operators of crossover, mutation and selection to evolve from a randomly generated initial population to a final solution.

The contribution of the current work is to present an alternative approach of DE combined with chaos sequences, called Chaotic Differential Evolution (CDE) for the optimization of electromagnetic devices. The potential of CDE is demonstrated on the optimization of a seventeen-parameter loudspeaker model. Comparative results against other evolutionary algorithms demonstrate the good performance of the CDE.

II. DIFFERENTIAL EVOLUTION

The fundamental idea behind DE is a scheme whereby it generates the trial parameter vectors. In each step, DE mutates vectors by adding weighted, random vector differentials to them. If the performance of the trial vector is better than that of the target, the target vector is replaced by the trial vector in the next generation. The variant implemented here was the DE/*rand*/1/*bin* [1]. In this case, the classical equation of DE is:

$$z_i(t+1) = x_{i_1}(t) + MF\cdot[x_{i_2}(t) - x_{i_3}(t)] \qquad (1)$$

In (1), i =1,2,...,N is the individual's index of population; t is the time (generation); $x_i(t) = \left[x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)\right]^{\mathrm{T}}$ stands for the position of the i-th individual of the population of N real-valued n-dimensional vectors; $z_i(t) = \left[z_{i1}(t), z_{i2}(t), \ldots, z_{in}(t)\right]^{\mathrm{T}}$ stands for the position of the i-th individual of a *mutant vector*; $MF > 0$ is a real parameter, called *mutation factor*, which controls the amplification of the difference between two individuals to avoid search stagnation. The mutation operation randomly select the target vector $x_{i1}(t)$, with $i \neq i_1$.Then, two individuals $x_{i2}(t)$ and $x_{i3}(t)$ are randomly selected with $i_1 \neq i_2 \neq i_3 \neq i$, and the difference vector x_{i2} - x_{i3} is calculated.

One of the simplest dynamic systems evidencing chaotic behavior is called *logistic map* [2], whose equation is given by:

$$y(t) = \mu \cdot y(t-1)\cdot\left[1 - y(t-1)\right] \qquad (2)$$

where t is the sample, and μ is a control parameter in the range $0 \leq \mu \leq 4$. The behavior of the system defined by (1) is highly sensitive to variations of μ. The value of μ determines whether y stabilizes at a constant size, oscillates between a limited sequence of sizes, or behaves chaotically in an unpredictable pattern. Very small differences in the initial value of y can cause substantial differences in its long-time behavior.

Eq. (2) is deterministic, displaying chaotic dynamics when $\mu = 4$ and $y(1) \notin \{0, 0.25, 0.50, 0.75, 1\}$. In our case, $y(t)$ is distributed in the range (0,1) provided the initial $y(1) \in (0,1)$. Fig.1 shows, only for example, the behavior of y for μ equals to 4, 3 and 1 with respect to the iteration (t) for a fixed value of y(0). It is easy to observe that the sequences are very different, depending on the value of the parameter μ .

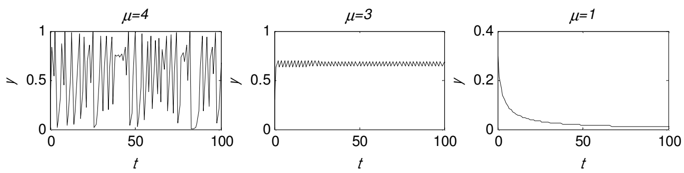

Fig. 1. The influence of the parameter μ

So, this work proposes a DE approach, which combines standard DE with chaotic sequences based on logistic map. In the DE context, the concepts of chaotic optimization methods can be useful. The parameters CR and MF of classical DE approaches are the key factors affecting the algorithm's convergence. However, both parameters cannot ensure the optimization's ergodicity completely in the search phase, since they are constant factors in traditional DE. Therefore, this paper offers a approach that introduces chaotic mapping with ergodicity, irregularity and the stochastic property into DE. This is done in order to improve the global convergence characteristics of the algorithm. The utilization of chaotic sequences in evolutionary algorithms can be useful to help the algorithms to escape from local minima.

The proposed modification adopted in CDE is given by:

$$z_i(t+1) = x_{i_1}(t) + y_i(t) \cdot [x_{i_2}(t) - x_3(t)] \ . \qquad (3)$$

III. LOUDSPEAKER PROBLEM

This model was based in the loudspeaker layout used as a test problem in [3]. The model is well described in [4], so it will omitted here. In this paper, we have considered the problem of minimizing the volume of material used in the construction of the loudspeaker. The device must also present at least some minimum prescribed value for the magnetic flux density in the air gap, in order to allow the loudspeaker to work properly. This condition was modeled as an inequality constraint, in which the value of $\boldsymbol{B}$ was required to be larger than a given fixed value. So, the proposed mathematical definition for the loudspeaker optimization problem used is:

$$\begin{aligned} &\min f(x) = \text{Volume} \\ &\text{Subject to: } |\boldsymbol{B}| \geq B_{ref} \end{aligned} \qquad (4)$$

The volume of material used in the loudspeaker is given by the solid generated by the revolution of the model around the central axis. The loudspeaker was modeled [5] using the LUA scripting language [6] to describe the geometry of the devices, and using the Finite Element Method Magnetics 4.0 (FEMM) [7] software as the magnetic solver.

IV. OPTIMIZATION RESULTS

For the optimization of loudspeaker problem, 30 independent runs were performed for each optimization method. The setup of DE and CDE approaches used was the following: crossover rate of CR = 0.8, population size equal to 15, and stopping criterion t_{max} = 400 generations. A constant mutation factor of MF = 0.4 was used in the classical DE approach.

The optimization results of evolution strategies (ES) [8] and particle swarm optimization [9] are also presented, in order to allow a comparative analysis of the performance of the proposed algorithm. The same number of evaluations of the objective (cost) function used by the DE and CDE approaches (15 x 400 = 6,000 evaluations) was used as the stopping criterion for the ES and PSO. Other particular parameters and procedures used in these optimization methods were:

- PSO: number of particles M = 15, social and cognitive coefficients both set as 2.05, and the inertia weight of each particle is linearly decreased over the course of each run, starting from 0.9 and ending at 0.4.
- ES: uses the sum strategy $ES(\mu+\lambda)$, where the number of parents and offspring are set to μ = 5 and $\lambda = 15$, respectively.

From Table I, one can see that the solutions of minimization of volume (minimum and mean values) obtained by DE and CDE are superior to the results of ES and PSO, with the best results being obtained by the CDE. DE was also able to obtain 'best run' results, but with a slightly worse performance than the one found by CDE. However, CDE was also the most robust algorithm for this problem, with the smallest "worst run" and standard deviation for the volume.

TABLE I
RESULTS OBTAINED FOR THE LOUDSPEAKER DESIGN

Parameter	DE	CDE	ES	PSO
Best run: Volume [mm^3]	5543.1	5406.5	11426.6	6431.8
Worst run: Volume [mm^3]	24890.4	6943.8	30757.4	37377.5
Average: Volume [mm^3]	6587.2	5906.7	22333.6	12059.3
Std. Dev.: Volume [mm^3]	5003.8	404	6425.9	8153
Rate of feasible solutions found (%)	100	100	46.67	100

V. CONCLUSION

In this paper a CDE technique for the global optimization of electromagnetic devices was introduced. The CDE algorithm takes advantage of the powerful characteristics of the DE methodology, with a modification in the routines for the generation of new candidate solutions to include diversity-promoting chaotic sequences based on the logistic map equation.

The comparative results obtained for an ES and a PSO approach show the potential of the CDE to obtain high-quality solutions to multimodal, nonlinear, and constrained problems in electromagnetics, even under a modest computational budget.

REFERENCES

[1] R. Storn and K. Price, "Differential evolution - a simple and efficient heuristic for global optimization over continuous spaces," *Journal of Global Optimization*, vol. 11, no. 4, pp. 341-359, 1997.

[2] R. May, "Simple mathematical models with very complicated dynamics," *Nature*, vol. 261, pp. 459-467, 1976.

[3] K. Rashid, J. A. Ramírez and E. M. Freeman, "Optimization of electromagnetic devices using computational intelligence techniques," *IEEE Transactions on Magnetics*, vol. 35, no. 5, pp. 3727-3729, 1999.

[4] F. Campelo, "Study on immune algorithms for the optimization of electromagnetic systems," *Master's thesis*, Graduate School of Information Science and Technology, Hokkaido University, Japan, 2006.

[5] F. Campelo, *http://www.cpdee.ufmg.br/~fcampelo/EN/files.html*

[6] R. Ierusalimschy, W. Celes, and L. H. Figueiredo, "LUA Scripting Language," [online]. Available from internet: *http://www.lua.org/*

[7] D. Meeker, "Finite Element Method Magnetics 4.0," [online]. Available from internet: *http://femm.foster-miller.net/*

[8] T. Bäck and H. -P. Schwefel, "An overview of evolutionary algorithms for parameter optimization," *Evolutionary Computation*, vol. 1, no. 1, pp. 1-23, 1993.

[9] R. C. Eberhart and J. Kennedy, "A new optimizer using particle swarm theory," in *Proceedings of the 6th International Symposium on Micro Machine and Human Science*, Nagoya, Japan, pp. 39-43, 1995